

Liquidity Risk

Size does matter

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REACH alternative
investment
management



Hedge Fund

RE3 Hedge Fund utility ?

RE3 Optimal Capital allocation

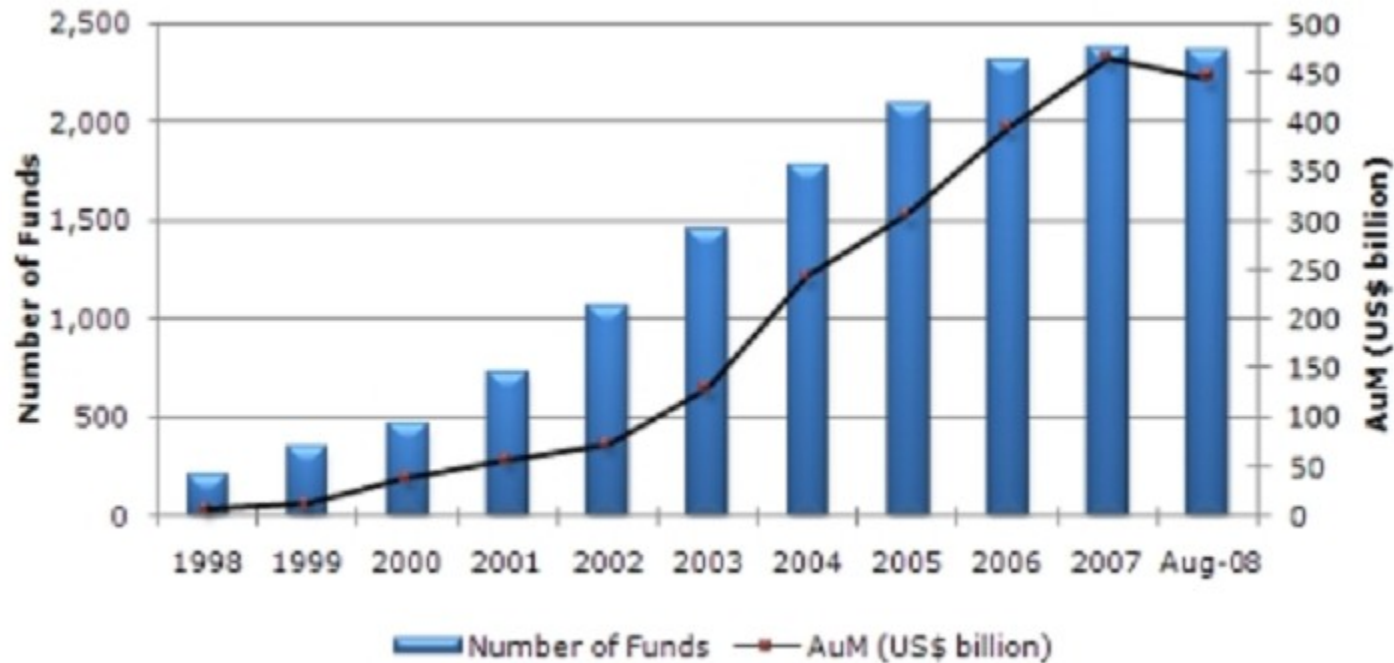
- Diversification
- Arbitrage
- New risk profile

RE3 Liquidity provider

- To the Market
- To the investors



AUM & Actors



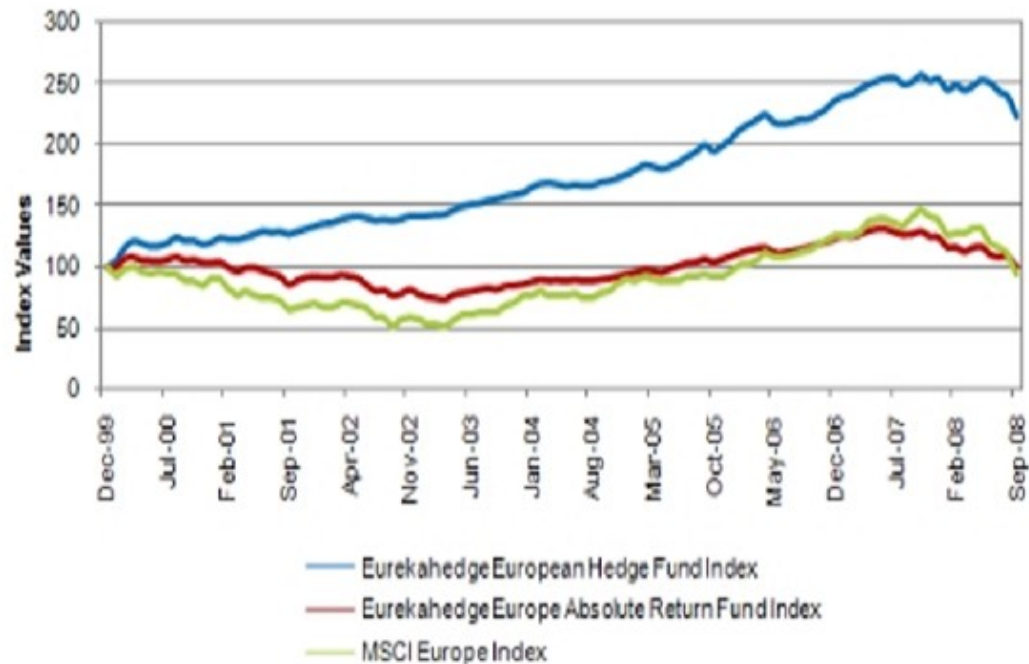
Source: EurekaHedge

- A steady increase of AUM and of number of HF until 2007...the crisis changes the picture



Performance

Index Performance Comparison



Source: Eureka hedge

ES The decorrelation of the HF performance with the indexes in question



Alternative investment and liquidity crisis

EE The financial crisis impacts the HF industry

EE A performance issue

- Weak and correlated performance
- Limited number of strategies
- Small capacity wrt performance impact

EE Poor liquidity

- Limited financing facility
- Illiquidity of the underlying

EE Investors on hold

- Fall in AUM
- Investors raise their standards



RE3 Hedge fund are

- Long correlation in stable market and short the systemic correlation
- Long the spread of liquidity between investors - market

RE3 Standard Liquidity indicators

- Market impact
- Number of days to close the positions

RE3 Features to manage the liquidity

- Lock up
- Gates...

RE3 Unfortunately the set up of the fund have been made according to

- Market practice : Lock up
- Emergency : Gates

RE3 But not wrt the “real” liquidity risk



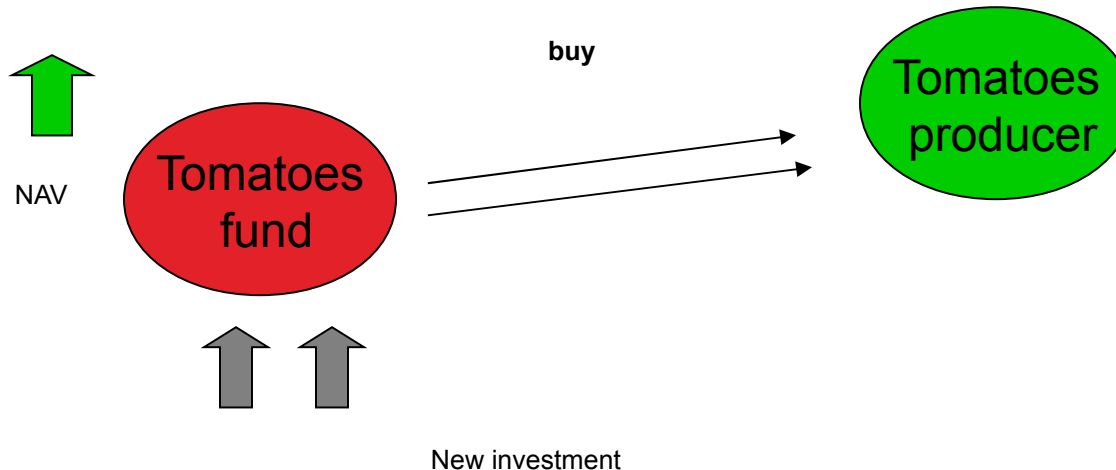
An asymmetric risk

EE A toy example

EE Fund with a stock X in illiquid asset

- Buy an extra x of asset \rightarrow move the price up by $y\%$ \rightarrow NAV of fund $+ y\%$
- Sell x of asset \rightarrow move the price down by $-y\%$ \rightarrow NAV of fund $- y\%$

EE A liquidity trap



EE It is always easier to buy than to sell



A simple framework

Investors Liquidity

Model of investors portfolio

- Each investors is ranked wrt its category, size of investment and probability to invest or redeem

Existing investors	Avg AUM per name	nbnames	Proba Redem 1M
INSTITUTIONS	1,000,000	30	25%
FUND OF HF	1,000,000	30	25%
FAMILY OFFICE	1,000,000	25	25%
HNWI	1,000,000	15	25%

Pipeline	AUM per name	nbnames	Proba Invest
INSTITUTIONS	1,000,000	2	80%
FUND OF HF	1,000,000	0	80%
FAMILY OFFICE	1,000,000	1	80%
HNWI	1,000,000	2	80%

today AUM : 100 M\$, new potential investors : 5 M\$



A simple framework

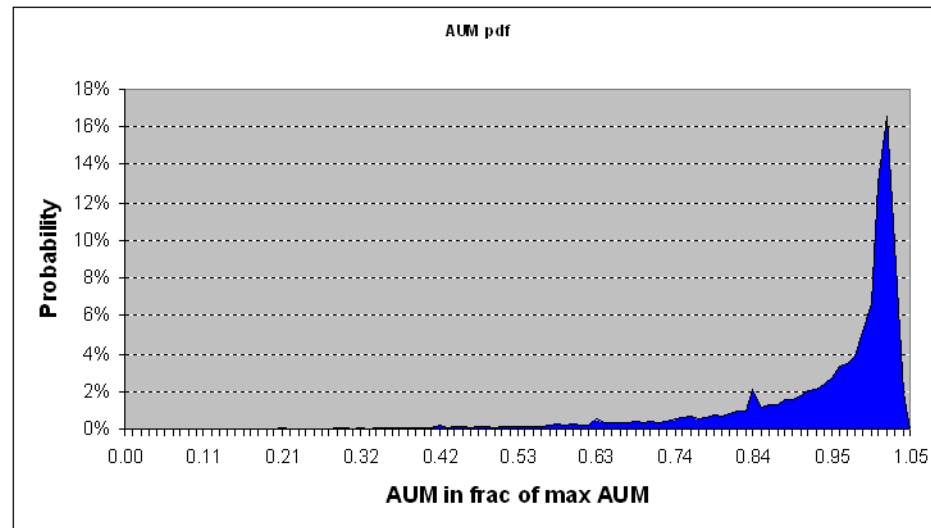
Model : simple copula with three parameters

- One correlation intra sector
- One correlation extra sector
- One correlation existing / new investors

Rho Intra Sector	0.5
Rho Extra Sector	0.4
Rho Old/New	-0.5

At a given date (1M or according to fund liquidity) we get the pdf of the AUM

Avg AUM : 94 M\$



Portfolio model

- One risky and non risky asset, no rate and dividend, simple BS model

$$dX = \theta \frac{dS}{S}, \quad X_0 = x$$

$$\frac{dS}{S} = \mu dt + \sigma dW$$

One period model

- At the end of the period the AUM is impacted by the redemption and the new investment

$$X \rightarrow X\varepsilon(\omega)$$

- Rebalancing without impacting the portfolio risk profile

$$\theta(\varepsilon(\omega) - 1)$$



Allocation model

- Cost of rebalancing according to an average liquidity L

$$\beta(\theta(\varepsilon(\omega) - 1) - L)_+$$

- Optimal portfolio

- Utility function

$$\theta = \text{ArgMin } EU(X, \varepsilon(\omega))$$

- Special case

$$\ln U(x, \varepsilon) = -\lambda x + \beta(\theta(\varepsilon - 1) - L)_+$$



Intuitive approach

⇒ Liquidity option

$$\ln EU(X, \varepsilon) = -\lambda\theta T + \frac{1}{2} \lambda^2 \theta^2 \sigma^2 T + \beta \text{Call}_L(\theta(\varepsilon - 1))$$

⇒ The optimal allocation accounts for the hedge of the option

$$\theta = \theta^* - c \text{Delta}_L(\theta(\varepsilon - 1)) \leq \theta^* := \frac{\mu}{\lambda \sigma^2}$$



Simple model result

Target allocation in risky asset 55%

- Beta : 10%
- L : 20 %

% Change in risky asset allocation

		Correlation		
		0	0.5	0.8
Proba redemption	10	0.0%	-4.6%	-11.3%
	20	0.0%	-14.6%	-24.3%
	30	-2.7%	-25.8%	-37.4%

If the proba of redemption increases, the investment in risky asset should decrease

If the investors are “correlated”, the investment in risky asset should decrease



Combined model

- ☒ N agents, they “share” the liquidity option

$$U(X, \varepsilon) = -\lambda X + \beta \text{Call}_L \left(\frac{1}{N} \sum_i \theta_i (\varepsilon_i - 1) \right)$$

- ☒ Two extreme cases for 2 agents (Proba redemption : 20, corr : 50%)

☒ Same investors : **- 14.6%**

☒ Independent investors : **- 7,9%**

- ☒ Intuitive result : at the limit, if the investors are “random”, almost no impact but if the investors are “shared” the risk is huge



Conclusion

- ▣ The HF industry moved from “random” or “positive” flow of AUM to highly correlated outflows

- ▣ It is crucial to quantify and manage the investors risk

- ▣ Key points
 - Better knowledge of investors
 - Diversify the strategies
 - Don't be short of liquidity option

- ▣ Extensions
 - Investors redemption / fund performance correlation
 - Multi period
 - Define optimal liquidity of the fund (lock up, gates)

